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Eigenvalue Approach to Study the Effect of Rotation and Relaxation Time in Generalised Thermoelasticity

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Abstract—The fundamental equations of the problems of generalized thermoelasticity with one relaxation parameter including heat sources in infinite rotating media have been written in the form of a vector-matrix differential equation in the Laplace transform domain for a one-dimensional problem. These equations have been solved by the eigenvalue approach. The results have been compared to those available in the existing literature. The graphs have been drawn to show the effect of rotation.
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1. INTRODUCTION

The governing equations for displacement and temperature fields in the linear dynamical theory of classical thermoelasticity consist of the coupled partial differential equation of motion and the Fourier's law of heat conduction equation. The equation for displacement field is governed by a wave type hyperbolic equation, whereas that for the temperature field is a parabolic diffusion type equation. This amounts to the remark that the classical thermoelasticity predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances, which are coupled together. This means that a part of every solution of the equations extends to infinity; cf. [1]. Experimental investigations by Ackerman *et al.* [2–4], von Gutfeld and Nethercot [5], Taylor *et al.* [6], Jackson and Walker [7], and many others, conducted on different solids, have shown that heat pulses do not propagate with infinite speeds. In order to overcome this paradox, efforts were made to modify classical thermoelasticity, on different grounds, for obtaining a wave type heat conduction equation; cf. [8–12], and others. A comprehensive list on this generalization for the last two decades is available in the work of Chandrasekharaiah [13].

At present there are two generalizations of the classical theory of elasticity: the first proposed by Lord and Shulman [1] (L-S theory) involves one relaxation time for a thermoelastic process,

and the second by Green and Lindsay [10] (G-L theory), which takes into account two parameters on relaxation times. Owing to the mathematical complexities encountered in coupled thermoelasticity, mainly due to inertia and coupling terms in governing equations, these types of problems are mostly confined to one-dimensional problems; cf. [14–18], and many others.

It appears that little attention has been paid to study the propagation of plane thermoelastic waves in a rotating medium. Since most large bodies like the earth, the moon, and other planets have an angular velocity, it appears more realistic to study the propagation of plane thermoelastic or magneto-thermoelastic waves in a rotating medium with relaxation.

However, in the present paper following one parameter L-S theory, the authors have considered a problem of rotation, in generalized theory of thermoelasticity by taking into account both the dynamic effect and the influence of coupling terms. Although this is a one-dimensional problem with heat sources distributed over a plane area in an infinite isotropic elastic solid, it involves two displacement components due to rotation. The new eigenvalue approach of Das *et al.* [17] has been applied for the solution of the problem.

2. BASIC EQUATIONS AND FORMULATION OF THE PROBLEMS

An infinite isotropic, homogeneous, thermally conducting elastic medium with density ρ and Lamé constants λ and μ is considered. The medium is rotating uniformly with an angular velocity $\vec{\Omega} = \Omega \vec{n}$, where \vec{n} is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms:

- (i) centripetal acceleration $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ due to time-varying motion only;
- (ii) the Coriolis acceleration $2\vec{\Omega} \times \dot{\vec{u}}$.

Here \vec{u} is the dynamic displacement vector measured from a steady-state deformed position and supposed to be small. These two terms do not appear in the equations for nonrotating media.

The equations of thermoelasticity proposed by (L-S theory) are

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad (2.1)$$

$$q_i = -k\theta_{,i}, \quad (2.2)$$

$$q_{i,i} = \rho c_v (\dot{\theta} + \alpha^* \ddot{\theta}) + \gamma \theta_0 \dot{\Delta}, \quad (2.3)$$

$$\tau_{ij} = \lambda \Delta \delta_{ij} + 2\mu_{ij} - \gamma (\theta + \alpha \dot{\theta}) \delta_{ij}, \quad (2.4)$$

where

$$\Delta = u_{i,i}, \quad \gamma = (3\lambda + 2\mu)\alpha_t.$$

The equations of motion in the absence of body forces are

$$\tau_{ij,j} = \rho \left[\ddot{u}_i + \left\{ \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) \right\}_i + \left(2\vec{\Omega} \times \dot{\vec{u}} \right)_i \right], \quad (2.5)$$

where all the terms have the same significance as in [1]. Combining (2.2) and (2.3), we obtain

$$k\theta_{,ij} = \rho c_v + \gamma \theta_0 \dot{\Delta} - \left(1 + \tau_0 \frac{\partial}{\partial t} \right) Q. \quad (2.6)$$

Combining (2.1), (2.4), and (2.5), we obtain the displacement equation of motion in the rotating frame of reference as

$$\rho \left[\ddot{\vec{u}} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega} \times \dot{\vec{u}} \right] = (\lambda + \mu) \nabla (\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} - \gamma \nabla (\theta + \alpha \dot{\theta}). \quad (2.7)$$

To write equations (2.6) and (2.7) in nondimensional forms, we define

$$\begin{aligned}\omega^* &= c_1^2 \delta, & c_1^2 &= \frac{\lambda + 2\mu}{\rho}, & g &= \frac{\gamma}{\rho c_v}, & \beta^2 &= \frac{\lambda + 2\mu}{\mu}, \\ b &= \frac{\gamma \theta_0}{\mu}, & \delta &= \frac{\rho c_v}{k}, & \Omega' &= \frac{\Omega}{\omega^*}, & Q &= \frac{Q'}{k \theta_0 \delta^2 c_1^2}.\end{aligned}$$

We also use the same letter to denote the nondimensional quantities. Thus, in what follows, $u_i, t, \theta, x_i, \alpha, \Omega$ will represent $c_1 \delta u_i, t \omega^*, \theta / \theta_0, c_1 \delta x_i, \alpha \omega^*, \Omega \omega^*$, respectively.

Let us consider a homogeneous isotropic thermoelastic solid occupying the whole space: $-\infty \leq x \leq \infty$, whose state depends only on the space variable x and the time variable t in dimensional form.

Since we are dealing with an isotropic solid medium, without any loss of generality, we may consider waves propagating in the x -direction. All field variables are supposed to be functions of x and t only. We consider the time varying dynamic solutions, and as such the time-independent part of the centripetal acceleration is neglected. We write $\vec{u} = (u, v, w)$.

In order to examine the effect of rotation and relaxation times on coupled elastic dilatational, shear, and thermal waves, we set $\vec{\Omega} = (0, 0, \Omega)$ where Ω is a constant. In view of these assumptions, equations (2.6) and (2.7) reduce to

$$\theta'' = \dot{\theta} + \tau_0 \ddot{\theta} + g(\dot{u}' + \tau_0 \ddot{u}') - (Q + \tau_0 \dot{Q}), \quad (2.8)$$

$$\beta^2 u'' - b \theta' = \beta^2 (\ddot{u} - \Omega^2 u - 2\Omega \dot{v}), \quad (2.9)$$

$$v'' = \beta^2 [\ddot{v} - \Omega^2 v + 2\Omega \dot{u}], \quad (2.10)$$

$$w'' = \beta^2 \ddot{w}, \quad (2.11)$$

where the primes and dots denote differentiation with respect to x and t , respectively. Equations (2.8)–(2.10) form a coupled system and represent coupled thermal, dilatational, and shear waves, while equation (2.11) uncouples from the system. The thermal field affects the shear motion due to rotation. This coupling disappears when $\Omega = 0$ and resulting equations coincide with those obtained by Sherief [19] and Das *et al.* [20] except that the uncoupled equations (2.10) and (2.11) do not appear in the discussion.

The constitutive relation (2.4) in the present analysis takes the form

$$\sigma = \beta^2 \frac{\partial u}{\partial x} - b \theta \quad (2.12)$$

in nondimensional form.

We assume that the heat source acts on the plane $x = 0$, and is of the form

$$Q = Q_0 \delta(x) H(t),$$

where Q_0 is a constant, and $\delta(x)$ and $H(t)$ are Dirac delta function of x and the Heaviside unit step function of t , respectively.

For the solution of these equations, we now apply the Laplace transform of parameter p defined by

$$\bar{u}(x, p) = \int_0^\infty u(x, t) \exp(-pt) dt, \quad \bar{v}(x, p) = \int_0^\infty v(x, t) \exp(-pt) dt,$$

and

$$\bar{\theta}(x, p) = \int_0^\infty \theta(x, t) \exp(-pt) dt. \quad (2.13)$$

Taking the Laplace transform on both sides of equations (2.8)–(2.10) and (2.12), we obtain

$$\frac{d^2\bar{\theta}}{dx^2} = p(1 + \tau_0 p) \left[\bar{\theta} + g \frac{du}{dx} - Q_0 \frac{\delta(x)}{p^2} \right], \quad (2.14)$$

$$\frac{d^2\bar{u}}{dx^2} - a \frac{d\bar{\theta}}{dx} = p^2\bar{u} - \Omega^2\bar{u} - 2\Omega\bar{v}, \quad (2.15)$$

$$\frac{d^2\bar{v}}{dx^2} = \beta^2 [(p^2 - \Omega^2)\bar{v} + 2p\Omega\bar{u}], \quad (2.16)$$

and

$$\bar{\sigma} = \beta^2 \frac{d\bar{u}}{dx} - b\bar{\theta}, \quad (2.17)$$

where $a = b/\beta^2$ and $\int_0^\infty \exp(-pt)\delta(t)dt = 1$ and $\int_0^\infty \exp(-pt)H(t)dt = 1/p$.

In deriving the previous results, use has been made of the conditions that u , $\frac{\partial u}{\partial t}$, and θ are initially zero, while $\bar{\theta}$, $\frac{\partial \bar{\theta}}{\partial x}$, and $\frac{\partial \bar{u}}{\partial x}$ are zero at infinity.

Following the method adopted by Das *et al.* [20], equations (2.14)–(2.16) can be written in the form of a vector matrix differential equation as follows:

$$\frac{d}{dx} \begin{bmatrix} \bar{\theta}(x, p) \\ \bar{u}(x, p) \\ \bar{v}(x, p) \\ \bar{\theta}'(x, p) \\ \bar{u}'(x, p) \\ \bar{v}'(x, p) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ c_{41} & 0 & 0 & 0 & c_{45} & 0 \\ 0 & c_{52} & c_{53} & c_{54} & 0 & 0 \\ 0 & c_{62} & c_{63} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\theta}(x, p) \\ \bar{u}(x, p) \\ \bar{v}(x, p) \\ \bar{\theta}'(x, p) \\ \bar{u}'(x, p) \\ \bar{v}'(x, p) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -Q_0\delta(x) \frac{(1 + \tau_0 p)}{p} \\ 0 \\ 0 \end{bmatrix}, \quad (2.18)$$

where

$$\begin{aligned} c_{41} &= p(1 + \tau_0 p), & c_{45} &= gc_{41}, & c_{52} &= p^2 - \Omega^2, & c_{53} &= -2\Omega, \\ c_{54} &= a, & c_{62} &= 2p\Omega\beta^2, & c_{63} &= \beta^2(p^2 - \Omega^2). \end{aligned} \quad (2.19)$$

The matrix differential equation (2.18) is the most general one as it includes rotation and is different from [20] where the order of the matrix is only four and the number of displacement components is one less than the present problem. Furthermore, if $\Omega = 0$, the relations (2.19) and the present equation (2.18) will reduce to those obtained in [20].

Equation (2.18) can be compactly written as

$$\frac{d}{dx} \tilde{V}(x, p) = \tilde{A}(p)\tilde{V}(x, p) + \tilde{B}(x, p), \quad (2.20)$$

where

$$\begin{aligned} \tilde{V}(x, p) &= [\bar{\theta}(x, p), \bar{u}(x, p), \bar{v}(x, p), \bar{\theta}'(x, p), \bar{u}'(x, p), \bar{v}'(x, p)]^T, \\ \tilde{B}(x, p) &= \left[0, 0, 0, -\frac{Q_0(1 + \tau_0 p)\delta(x)}{p}, 0, 0 \right]^T, \\ \tilde{A}(p) &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ c_{41} & 0 & 0 & 0 & c_{45} & 0 \\ 0 & c_{52} & c_{53} & c_{54} & 0 & 0 \\ 0 & c_{62} & c_{63} & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (2.21)$$

Equation (2.18) is to be solved subject to the conditions

$$\begin{aligned} \text{(i)} \quad & \theta = u = \frac{\partial u}{\partial t} = 0, \quad \text{at } t = 0, \quad \forall x, \\ \text{(ii)} \quad & \bar{\theta} = \bar{u} = \frac{\partial \bar{\theta}}{\partial x} = \frac{\partial \bar{u}}{\partial x} = 0, \quad \text{at } x = \pm\infty, \quad \forall t > 0. \end{aligned} \quad (2.22)$$

3. SOLUTION OF THE VECTOR MATRIX DIFFERENTIAL EQUATION

Let us now proceed to solve equation (2.18) by the eigenvalue approach proposed by Das *et al.* [20]. The characteristic equation of the matrix $\tilde{A}(p)$ takes the form

$$\lambda^6 - \lambda^4 (c_{41} + c_{52} + c_{63} + c_{45}c_{54}) + \lambda^2 (c_{41}(c_{52} + c_{63}) + c_{52}c_{63} - c_{53}c_{62} + c_{45}c_{54}c_{63}) - c_{41}(c_{52}c_{63} - c_{53}c_{62}) = 0. \quad (3.1)$$

The roots of the characteristic equation (3.1) which are also eigenvalues of the matrix $\tilde{A}(p)$ may be written in the form

$$\lambda = \pm\lambda_1, \quad \lambda = \pm\lambda_2, \quad \lambda = \pm\lambda_3. \quad (3.2)$$

The right eigenvector $\vec{X} = [X_1, X_2, X_3, X_4, X_5, X_6]^T$, corresponding to the eigenvalue λ , can be written as

$$\vec{X} = \begin{bmatrix} -\lambda c_{45}(c_{63} - \lambda^2), & (c_{41} - \lambda^2)(c_{63} - \lambda^2), & -c_{62}(c_{41} - \lambda^2), \\ -\lambda^2 c_{45}(c_{63} - \lambda^2), & \lambda(c_{41} - \lambda^2)(c_{63} - \lambda^2), & -\lambda^3 c_{45}(c_{63} - \lambda^2) \end{bmatrix}^T. \quad (3.3)$$

From (3.2) and (3.3), we can easily calculate the eigenvector X_i , corresponding to the eigenvalue $\lambda = \lambda_i$, $i = 1, 2, 3, 4, 5, 6$. For further reference, we shall use the following notations:

$$\begin{aligned} X_1 &= [X]_{\lambda=\lambda_1}, & X_2 &= [X]_{\lambda=-\lambda_1}, & X_3 &= [X]_{\lambda=\lambda_2}, \\ X_4 &= [X]_{\lambda=-\lambda_2}, & X_5 &= [X]_{\lambda=\lambda_3}, & X_6 &= [X]_{\lambda=-\lambda_3}. \end{aligned} \quad (3.4)$$

The left eigenvector $Y = [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6]$ corresponding to the eigenvalue λ can be calculated as

$$\begin{aligned} Y &= [\lambda^2 c_{54}(c_{63} - \lambda^2) - c_{54}(c_{41} - \lambda^2)(c_{63} - \lambda^2), \lambda \{ (c_{41} - \lambda^2)(c_{63} - \lambda^2) \\ &\quad + c_{45}c_{54}(c_{63} - \lambda^2) \}, -\lambda c_{53}(c_{41} - \lambda^2) - \lambda c_{54}(c_{63} - \lambda^2), \\ &\quad (c_{41} - \lambda^2)(c_{63} - \lambda^2), -c_{53}(c_{41} - \lambda^2) \end{aligned} \quad (3.5)$$

and as discussed earlier, the left eigenvector Y_i , corresponding to the eigenvalue λ_i , $i = 1, 2, 3, 4, 5, 6$ may be denoted as

$$\begin{aligned} Y_1 &= [Y]_{\lambda=\lambda_1}, & Y_2 &= [Y]_{\lambda=-\lambda_1}, & Y_3 &= [Y]_{\lambda=\lambda_2}, \\ Y_4 &= [Y]_{\lambda=-\lambda_2}, & Y_5 &= [Y]_{\lambda=\lambda_3}, & Y_6 &= [Y]_{\lambda=-\lambda_3}. \end{aligned} \quad (3.6)$$

Considering other physical conditions of the problem as in [18], the solution of equation (2.18) can be written as

$$\tilde{V}(x, p) = a_2(x)x_2 e^{-\lambda_1 x} + a_4(x)x_4 \exp(-\lambda_2 x) + a_6(x)x_6 \exp(-\lambda_3 x), \quad x > 0, \quad (3.7)$$

where

$$\begin{aligned} a_2(x) &= \frac{1}{Y_2 X_2} \int_{-\infty}^x y_2 \tilde{B}(s, p) \exp(-\lambda_1 s) ds, \quad x > 0 \\ &= \frac{-\lambda_1 c_{54}(c_{63} - \lambda_1^2)}{Y_2 X_2} \cdot \frac{Q_0(1 + \tau_0 p)}{p}. \end{aligned} \quad (3.8)$$

Similarly,

$$a_4(x) = \frac{(-\lambda_2)c_{54}(c_{63} - \lambda_2^2)}{Y_4 X_4} \cdot \frac{Q_0(1 + \tau_0 p)}{p}, \quad (3.9)$$

$$a_6(x) = \frac{(-\lambda_3)c_{54}(c_{63} - \lambda_3^2)}{Y_6 X_6} \cdot \frac{Q_0(1 + \tau_0 p)}{p}. \quad (3.10)$$

Thus, the displacement and temperature fields can be written from (3.7) as

$$\begin{aligned}\bar{u}(x, p) = & a_2(x) [(c_{41} - \lambda_1^2) (c_{63} - \lambda_1^2)] \exp(-\lambda_1 x) \\ & + a_4(x) [(c_{41} - \lambda_2^2) (c_{63} - \lambda_2^2)] \exp(-\lambda_2 x) \\ & + a_6(x) [(c_{41} - \lambda_3^2) (c_{63} - \lambda_3^2)] \exp(-\lambda_3 x),\end{aligned}\quad (3.11)$$

$$\begin{aligned}\bar{v}(x, p) = & a_2(x) [-c_{62} (c_{41} - \lambda_1^2)] \exp(-\lambda_1 x) \\ & + a_4(x) [-c_{62} (c_{41} - \lambda_2^2)] \exp(-\lambda_2 x) \\ & + a_6(x) [-c_{62} (c_{41} - \lambda_3^2)] \exp(-\lambda_3 x),\end{aligned}\quad (3.12)$$

$$\begin{aligned}\bar{\theta}(x, p) = & a_2(x) [\lambda_1 c_{45} (c_{63} - \lambda_1^2)] \exp(-\lambda_1 x) \\ & + a_4(x) [\lambda_2 c_{45} (c_{63} - \lambda_2^2)] \exp(-\lambda_2 x) \\ & + a_6(x) [\lambda_3 c_{45} (c_{63} - \lambda_3^2)] \exp(-\lambda_3 x).\end{aligned}\quad (3.13)$$

Using equations (3.11) and (3.13), in (2.17), we write the stress component $\bar{\sigma}(x, p)$ as

$$\begin{aligned}\bar{\sigma}(x, p) = & (\beta^2 + b) \left[\frac{\lambda_1^2 (c_{63} - \lambda_1^2)^2 \{ (c_{41} - \lambda_1^2) + c_{45} \} \exp(-\lambda_1 x)}{Y_2 X_2} \right. \\ & + \frac{\lambda_2^2 (c_{63} - \lambda_2^2)^2 \{ (c_{41} - \lambda_2^2) + c_{45} \} \exp(-\lambda_2 x)}{Y_4 X_4} \\ & \left. + \frac{\lambda_3^2 (c_{63} - \lambda_3^2)^2 \{ (c_{41} - \lambda_3^2) + c_{45} \} \exp(-\lambda_3 x)}{Y_6 X_6} \right] c_{54} \cdot Q_0 \frac{(1 + \tau_0 p)}{p}.\end{aligned}\quad (3.14)$$

Writing (a_2, a_4, a_6) as (A_1, A_2, A_3) , the deformations $\bar{u}(x, p)$, $\bar{v}(x, p)$, the stress field $\bar{\sigma}(x, p)$, and the temperature field $\bar{\theta}(x, p)$ can be compactly written as

$$\begin{aligned}\bar{u}(x, p) = & \sum_{i=1}^3 A_i [(c_{41} - \lambda_i^2) (c_{63} - \lambda_i^2)] \exp(-\lambda_i x), \\ \bar{v}(x, p) = & \sum_{i=1}^3 A_i [-c_{62} (c_{41} - \lambda_i^2)] \exp(-\lambda_i x), \\ \bar{\theta}(x, p) = & \sum_{i=1}^3 A_i [\lambda_i c_{45} (c_{63} - \lambda_i^2)] \exp(-\lambda_i x), \\ \bar{\sigma}(x, p) = & \sum_{i=1}^3 A_i (\beta^2 + b) [\lambda_i (c_{63} - \lambda_i^2) \{ (c_{41} - \lambda_i^2) + c_{45} \} \exp(-\lambda_i x)].\end{aligned}\quad (3.15)$$

Equations (3.11)–(3.14) determine completely the state of the solid for $x > 0$. The solution for the whole space (when the space $x \leq 0$ is also included) is obtained from (3.11)–(3.14) by taking the symmetries under consideration. Thus, considering the heat source to act at the location $x = c$, instead of $x = 0$, we may write down the field variables as follows:

$$\begin{aligned}\bar{u}(x, p) = & - \left[\frac{\lambda_1^2 (c_{63} - \lambda_1^2)^2 \exp(\pm \lambda_1 (x - c))}{V_2} + \frac{\lambda_2^2 (c_{63} - \lambda_2^2)^2 \exp(\pm \lambda_2 (x - c))}{V_4} \right. \\ & \left. + \frac{\lambda_3^2 (c_{63} - \lambda_3^2)^2 \exp(\pm \lambda_3 (x - c))}{V_6} \right] \frac{c_{45} c_{54} Q_0 (1 + \tau_0 p)}{p}\end{aligned}\quad (3.16)$$

and three similar expressions for $\bar{v}(x, p)$, $\bar{\theta}(x, p)$, and $\bar{\sigma}(x, p)$ where $V_2 = Y_2 X_2$, $V_4 = Y_4 X_4$, and $V_6 = Y_6 X_6$ and the upper sign indicates the solution in the region $x \leq c$, while the lower sign

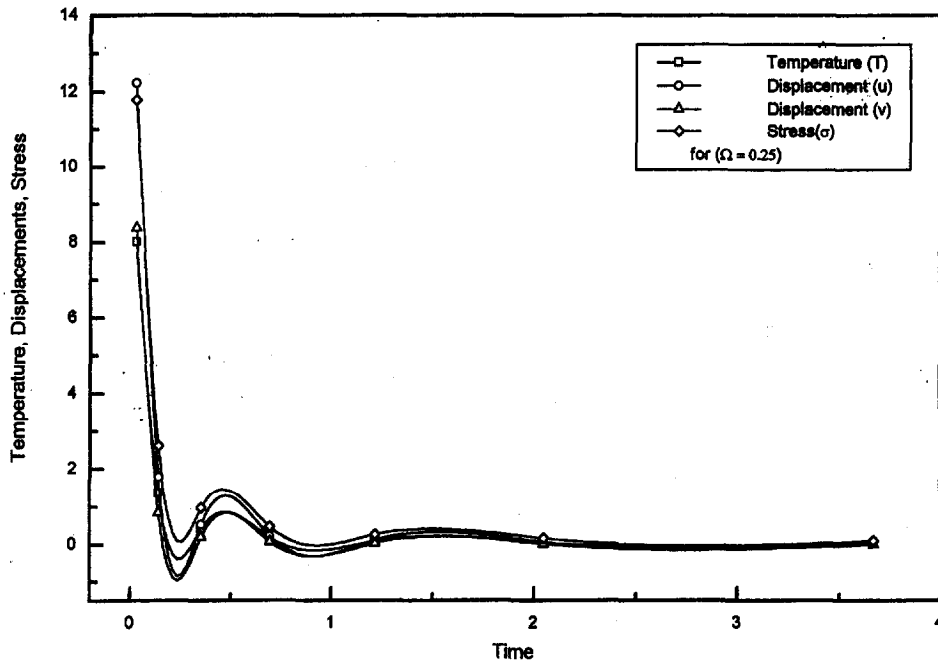


Figure 1. Effect of rotation on displacements, temperature, and stress for different values of time.

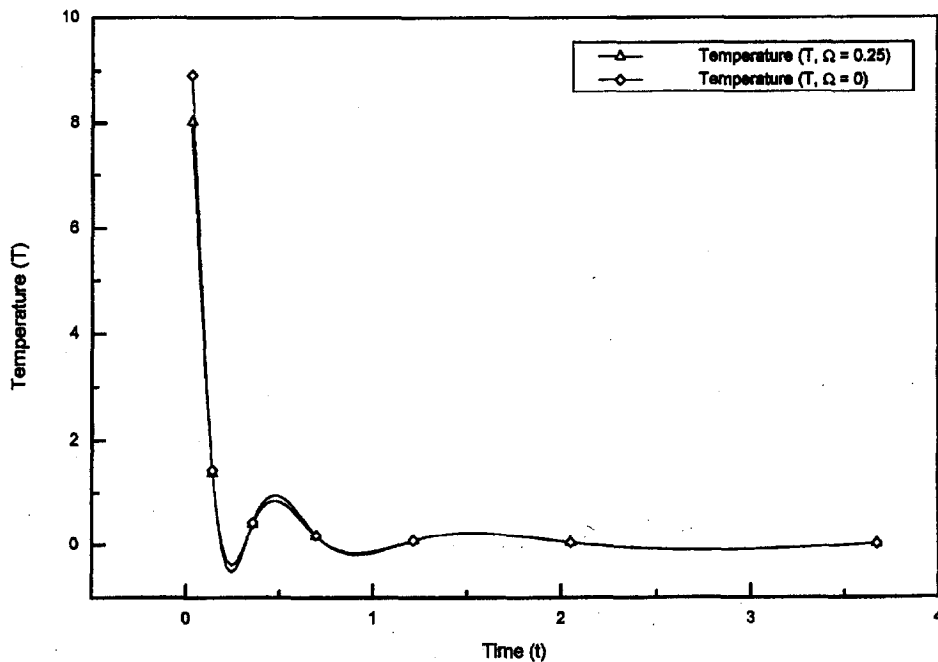


Figure 2. Effect of rotation on temperature.

indicates the solution in the region $x > c$. These results may be compared to those of Das *et al.* [20] and Sherief [19] except that in both these cases, rotation component is absent ($\Omega = 0$).

4. NUMERICAL SOLUTION AND DISCUSSION

Sherief [19] presented the solutions for two fixed values of time with varying space variable x when the rotation parameter is absent. The present authors prefer to determine the state in the

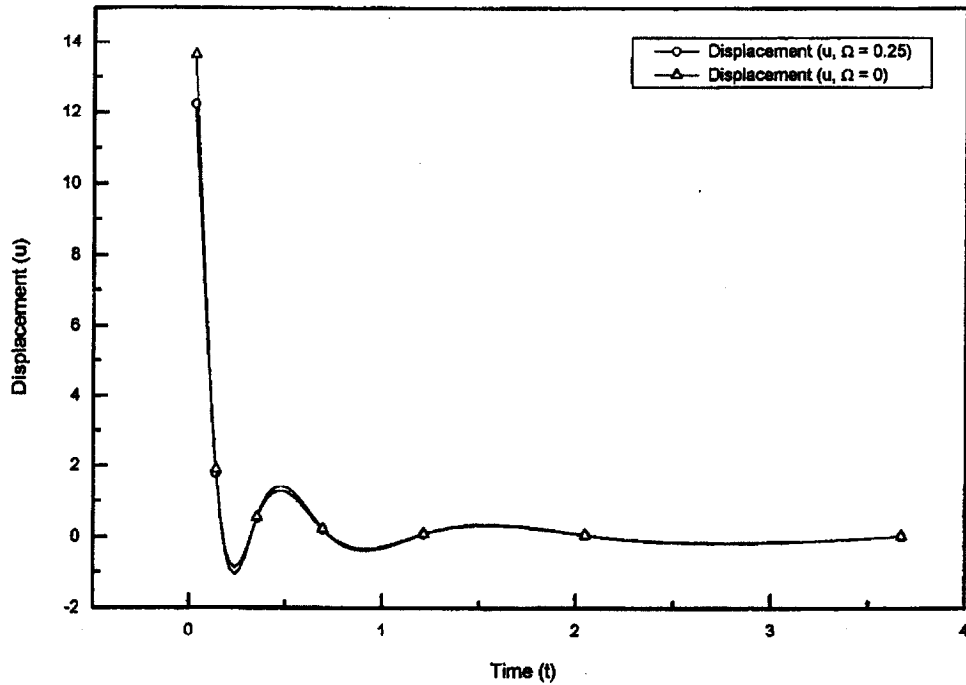
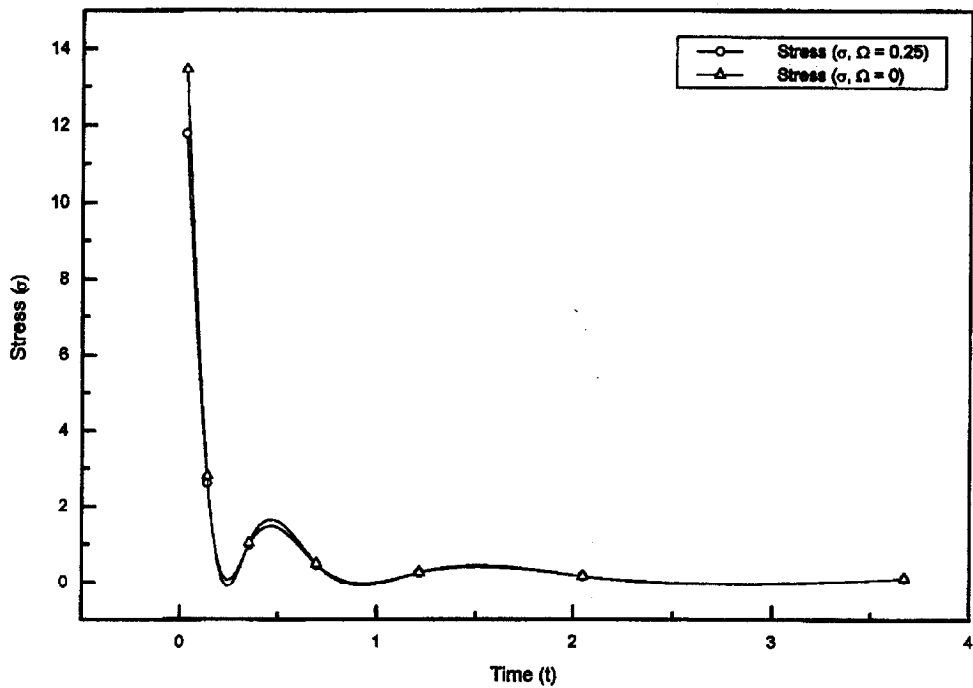
Figure 3. Effect of rotation on displacement (u).

Figure 4. Effect of rotation on stress.

space-time domain numerically by Bellman's [21] method for a fixed value of a space variable and for varying time. The computations for the state variables are carried out for a fixed value of $x = 1$ and for values of $t = 0.0257750, 0.138382, 0.352509, 0.693147, 1.21376, 2.4612, 3.67119$ which are also the roots of the Legendre polynomial of degree seven; cf. [21]. The copper material is

chosen for numerical computation. The values of the nondimensional constants ε , β^2 , τ_0 , a , b , g are taken as below,

$$a = 0.01344, \quad b = 0.04704, \quad g = 1.25, \quad \varepsilon = 0.0168, \quad \beta^2 = 3.5, \quad \tau_0 = 0.02 \text{ (sec.)}.$$

Using cubic spline formalism, the computed values of the state variables are then plotted in the graphs. The graphs for temperature ($T = T/aQ_0$), displacements ($u = u/aQ_0$, $v = v/aQ_0$), and stress ($\sigma = \sigma/a(b + \beta^2)$) are drawn for suitable values of Ω in Figure 1. The graphs for T , u , and σ are also drawn for $\Omega = 0.25$ and for $\Omega = 0$ in Figures 2–4, respectively, to show the effect of rotation. As it is expected that when $\Omega = 0$, the displacement v will be equal to zero, so no graph corresponding to v has been available for comparison. From the numerical results and graphs, it has been found that due to presence of rotation parameter, the numerical values of temperature, displacement, and stress have been decreased. In the course of computation, it has also been found that the roots of the characteristic equation (3.1) are real and pair-wise equal in magnitude and opposite in sign when $\Omega = 0$, and has the similar behaviour when $0 \leq \Omega \leq 0.25$. Some of the roots of equation (3.1) become complex when $0.25 < \Omega \leq 20$. When Ω increases beyond 20, then some of the roots of equation (3.1) become imaginary. To show the effect of Ω , and to avoid complications in calculation, only real values of λ are taken for an example of analysis.

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